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EXPERIMENTAL STUDY OF THE TURBULENT HEAT FLUX BALANCE COMPONENTS IN THE CROSS-SECTION OF A RETARDED TURBULENT BOUNDARY LAYER

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Abstract – The paper presents experimental data on distribution of the turbulent heat flux transport equation terms in the cross-section of an equilibrium retarded turbulent boundary layer. Comparison of individual terms of the equation with the known approximations is made.

NOMENCLATURE

B_1, B_2,	dimensionless constants;
c_p,	specific heat [J kg^{-1} K^{-1}];
c_f,	local friction coefficient;
e,	kinetic energy of turbulence [m^2 s^{-2}];
$F_{TT}(K)$,	normalized spectra of temperature fluctuations;
H,	$= \delta^*/\delta^{**}$, formparameter;
k,	thermal conductivity [W m^{-1} K^{-1}];
K,	wave number [m^{-1}];
Pr,	Prandtl number;
Pr_T,	turbulent Prandtl number;
T,	mean temperature [K];
U, V,	mean velocity components in x and y directions [m s^{-1}];
u', v',	velocity fluctuations in x and y directions [m s^{-1}];
x, y,	Cartesian coordinates [m];
t,	time [s];
R_{TT},	$= \dfrac{\overline{\theta'\theta'(y)}}{\sqrt{\overline{\theta'^2}}\sqrt{\overline{\theta'^2}}}$, correlation coefficient.

Greek symbols

Π,	$= \dfrac{\delta^*}{\tau_w}\dfrac{dP}{dx}$, pressure gradient parameter;
δ,	boundary layer thickness [m];
δ_T,	thermal boundary layer thickness [m];
δ^*,	displacement thickness [m];
δ^{**},	momentum thickness [m];
ρ,	density [kg m^{-3}];
θ',	temperature fluctuations [K];
ν,	viscosity [m^2 s^{-1}];
ν_T,	turbulent viscosity [m^2 s^{-1}];
τ,	shear stress [N m^{-2}].

Subscripts

∞,	free stream;
w,	wall.

Current calculations of heat transfer in a turbulent boundary layer involve different relations for the turbulent Prandtl number, Pr_T. In the simplest models, this number is assumed to be constant over the cross-section of the boundary layer. However, it is known [1, 2] that distribution of Pr_T depends on both the hydrodynamic and thermal boundary conditions. Therefore, in a number of interesting engineering problems involving flows with variable hydrodynamic and thermal boundary conditions, these models are sure to provide inaccurate results which fail to give an actual picture of the flow.

In Refs. [3–5], the method has been developed for closing the system of the boundary layer equations through a turbulent heat flux transport equation. This equation is obtainable from the motion and energy equations by the use of the general method of deriving balance equations [6]. For a steady-state two-dimensional turbulent boundary layer, this equation has the following form:

$$
\begin{aligned}
U\frac{\partial\overline{v'\theta'}}{\partial x} + V\frac{\partial\overline{v'\theta'}}{\partial y} = &-\left(\overline{u'v'}\frac{\partial T}{\partial x} + \overline{v'^2}\frac{\partial T}{\partial y}\right) \\
&-\left(\overline{u'v'}\frac{\partial v}{\partial x} + \overline{v'\theta'}\frac{\partial v}{\partial y}\right) \\
&-\frac{1}{\rho}\frac{\partial\overline{\theta' p'}}{\partial y} - \left(\frac{\partial\overline{u'v'\theta'}}{\partial x}\right. \\
&\left.+\frac{\partial\overline{v'^2\theta'}}{\partial y} + \frac{1}{\rho}\overline{p'\frac{\partial\theta'}{\partial y}}\right) \\
&+\frac{k}{\rho c_p}\overline{v'\nabla^2\theta'} + \nu\overline{\theta'\nabla^2 v'}.
\end{aligned} \tag{1}
$$

The LHS of equation (1) characterizes a convective transport of the quantity $\overline{v'\theta'}$ by an averaged flow.

The first and the second groups of terms on the RHS of this equation ascribe the 'generation' of $\overline{v'\theta'}$ to mean temperature and velocity gradients, respectively. The term $1/\rho\ \partial\overline{\theta' p'}/\partial y$ characterizes the coupling between the pressure and temperature fluctuations. The fourth group of terms represents the turbulent diffusion of the quantity $\overline{v'\theta'}$. The last two terms characterize 'dissipation' of $\overline{v'\theta'}$ due to thermal conductivity and viscosity.

Except for the convective term and the terms which characterize 'generation', the remaining terms of equation (1) are unknown quantities. To describe the unknown correlations appearing in equation (1), the authors of [3–5] make use of different approximating relations composed, in the main, on dimensional grounds. It is, therefore, of great interest to experimentally determine separate terms of the turbulent heat flux transport equation in the diffusor region of flow and to compare the results obtained with the available approximations.

The authors of the present work have experimentally determined the components of equation (1) over the cross-section of a highly retarded equilibrium turbulent boundary layer having the following parameters:

$$U_{,} \sim x^{-0.253};\ c_f = 8 \times 10^{-4};$$

$$H = \frac{\delta^*}{\delta^{**}} = 1.935;\ \Pi = \frac{\delta^*}{\tau_w}\frac{\mathrm{d}P}{\mathrm{d}x} = 14.5.$$

A detailed description of the experimental facility, mean and fluctuation characteristics of the boundary layer under study and the technique of their determination is given in [1, 7, 8].

Since the mean velocity and temperature profiles, and fluctuational hydrodynamic and temperature characteristics in cross-sections of the equilibrium boundary layer studied are, according to [1], similar respectively in the coordinates y/δ and y/δ_T, then the convective term may be rewritten in a form which is more convenient for experimental determination

$$U\frac{\partial\overline{v'\theta'}}{\partial x} + V\frac{\partial\overline{v'\theta'}}{\partial y} = U\frac{2\overline{v'\theta'}}{\Delta T_{,}}\frac{\mathrm{d}\,\Delta T_{,}}{\mathrm{d}x} - \left(\frac{1}{\delta}\frac{\mathrm{d}\delta}{\mathrm{d}x} + \frac{1}{U_{,}}\frac{\mathrm{d}U_{,}}{\mathrm{d}x}\right)\frac{\partial\overline{v'\theta'}}{\partial y}\int_0^{\delta} U\,\mathrm{d}y + U\left(\frac{y}{\delta}\frac{\mathrm{d}\delta}{\mathrm{d}x} - \frac{y}{\delta_T}\frac{\mathrm{d}\delta_T}{\mathrm{d}x}\right)\frac{\mathrm{d}\overline{v'\theta'}}{\mathrm{d}y}.$$

Unfortunately, the terms of equation (1), which account for the coupling between the pressure and temperature fluctuations, as well as the term $\partial\overline{u'v'\theta'}/\partial x$, have not been determined.

The most complex, as regards experimental determination, is the 'dissipative' term. In expanded form it is written as

$$\frac{k}{\rho c_p}\overline{v'\nabla^2\theta'} + \overline{\nu\theta'\nabla^2 v'} = \frac{k}{\rho c_p}\overline{v'\left(\frac{\partial^2\theta'}{\partial x^2} + \frac{\partial^2\theta'}{\partial y^2} + \frac{\partial^2\theta'}{\partial z^2}\right)} + \overline{\nu\theta'\left(\frac{\partial^2 v'}{\partial x^2} + \frac{\partial^2 v'}{\partial y^2} + \frac{\partial^2 v'}{\partial z^2}\right)}.$$

Out of the six instantaneous derivatives of the θ' and v' fluctuations only the spatial derivative with respect to x has been determined. We have utilized a 'conventional' method of determining the time derivative with the help of the block of differentiation with subsequent recalculation of the time coordinates into the spatial ones following Taylor's hypothesis

$$\frac{\partial}{\partial x} = -\frac{1}{U}\frac{\partial}{\partial t}.$$

The derivatives with respect to the coordinates y and z were calculated by the isotropic relations

$$\overline{\frac{\partial^2\theta'}{\partial x^2}} = \overline{\frac{\partial^2\theta'}{\partial y^2}} = \overline{\frac{\partial^2\theta'}{\partial z^2}};\quad \overline{\frac{\partial^2 v'}{\partial x^2}} = 2\overline{\frac{\partial^2 v'}{\partial y^2}} = \overline{\frac{\partial^2 v'}{\partial z^2}}.$$

In the final form, the 'dissipative' term may be represented as

$$\frac{k}{\rho c_p}\overline{v'\nabla^2\theta'} + \overline{\nu\theta'\nabla^2 v'} = 3\frac{k}{\rho c_p}\overline{v'\left[\frac{\partial^2\theta'}{\partial x^2}\right]} + 2.5\,\overline{\nu\theta'\left[\frac{\partial^2 v'}{\partial x^2}\right]}.$$

The remainder terms of (1) were determined directly with the use of a thermometer and analog devices of the DISA 55M system.

The components of equation (1), being nondimensionalized with the help of the scales U_T, $T_{,}$, δ_T, are given in Fig. 1. This figure also contains the value of disbalance determined experimentally. A relatively small disbalance with respect to the remainder terms of equation (1) attests to reliability of the method used and validity of the experimental data obtained. It should be noted that in contrast to the turbulence

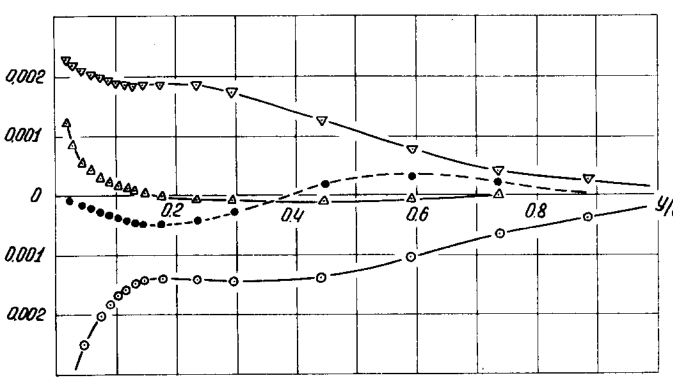

FIG. 1. Distribution of the turbulent heat flux balance components. ▽, 'dissipation'; △, diffusion; ○, 'production'; ●, disbalance.

energy [9], the diffusional turbulent transport of the quantity $\overline{v'\theta'}$ is important only in the wall region, $y/\delta_T < 0.1$.

The experimental data obtained have made it possible to check the approximations used. Usually, the turbulent diffusion is presented in the form of a gradient transfer

$$\left(\frac{\partial \overline{u'v'\theta'}}{\partial x} + \frac{\partial \overline{v'^2\theta'}}{\partial y}\right) \approx \frac{\partial}{\partial y}\left[\frac{\nu_T}{Pr_T}\frac{\partial}{\partial y}\overline{v'\theta'}\right]. \quad (2)$$

Figure 2 shows the comparison of the above approximation with the experimental data. One can easily see that there is only a qualitative agreement between the 'predicted' distribution and that obtained experimentally. However, when the diffusion (2) is given [3] as

$$\left(\frac{\partial \overline{u'v'\theta'}}{\partial x} + \frac{\partial \overline{v'^2\theta'}}{\partial y}\right) \approx B_1\frac{\partial}{\partial y}\left[\frac{\nu_T}{Pr_T}\frac{\partial}{\partial y}\overline{v'\theta'}\right] + B_2\frac{\sqrt{e}}{L_{uu}}\overline{v'\theta'} \quad (3)$$

then the results of calculation by (3) agree satisfactorily with the experimental data.

For 'dissipation' of the quantity $\overline{v'\theta'}$, the viscous processes governed by the term $\nu\overline{\theta'\nabla^2 v'}$ are of particular importance. To ease the choice of an approximating relation, let us transform the dissipative term to the form

$$\frac{k}{\rho c_p}\overline{v'\nabla^2\theta'} + \nu\overline{\theta'\nabla^2 v'} = \nu\overline{\theta'\nabla^2 v'}\left(\frac{1}{Pr}\frac{\overline{v'\nabla^2\theta'}}{\overline{\theta'\nabla^2 v'}} + 1\right).$$

Figure 3 presents the distribution of the quantity $\overline{v'\nabla^2\theta'}/\overline{\theta'\nabla^2 v'}$ accounting for contributions of viscosity and thermal conductivity to 'dissipation' of $\overline{v'\theta'}$. The analysis shows that thermal conductivity contributes but slightly to dissipation of this quantity. Even in the transition region of the turbulent boundary layer it

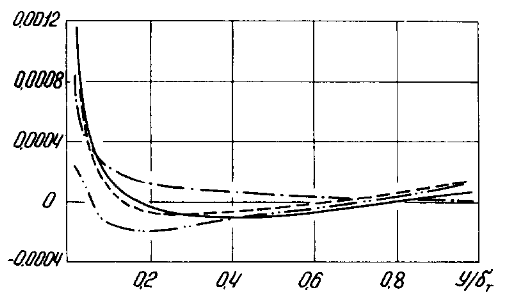

FIG. 2. Distribution of the diffusion of $\overline{v'\theta'}$.

———, experiment; —·—, $0.15\frac{\sqrt{e}}{L_{uu}}\overline{v'\theta'}$;

—··—, $4\frac{\partial}{\partial y}\left[\frac{\nu_T}{Pr_T}\frac{\partial}{\partial y}\overline{v'\theta'}\right]$;

— — —, $0.15\frac{\sqrt{e}}{L_{uu}}\overline{v'\theta'} + 4\frac{\partial}{\partial y}\left[\frac{\nu_T}{Pr_T}\frac{\partial}{\partial y}\overline{v'\theta'}\right]$.

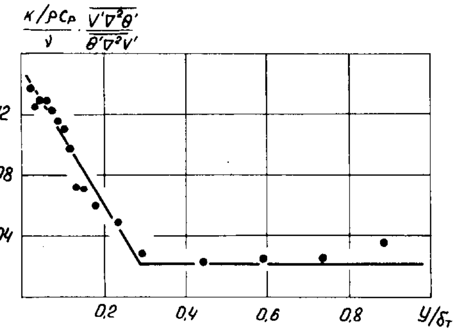

FIG. 3. Contributions of thermal conductivity and viscosity to 'dissipation' of $\overline{v'\theta'}$.

amounts to a little over 1%. Therefore this contribution may be neglected in the subsequent analysis. Noting experimental distribution of 'dissipation' of $\overline{v'\theta'}$ to be similar to distribution of the turbulent heat flux quantity itself, $\overline{v'\theta'}$, one may suppose that these quantities are proportional.

Thus, on dimensional grounds, we shall write 'dissipation' of the quantity $\overline{v'\theta'}$ in the following form:

$$\nu\overline{\theta'\nabla^2 v'}\left(\frac{1}{Pr}\frac{\overline{v'\nabla^2\theta'}}{\overline{\theta'\nabla^2 v'}} + 1\right) \approx \nu\frac{\overline{v'\theta'}}{\lambda_{TT}^2}.$$

In this expression

$$\lambda_{TT} = \sqrt{\frac{2\overline{\theta'^2}}{\overline{\left(\frac{\partial\theta'}{\partial x}\right)^2}}} = \sqrt{\frac{2}{\int_0^\infty K^2 F_{TT}(K)\mathrm{d}K}}$$

where λ_{TT} is the transverse dissipative scale.

Figure 4 gives the results of calculation of 'dissipation' of the quantity $\overline{v'\theta'}$ compared with the experimental data. There is a good correspondence between prediction and experiment. However, to determine the 'dissipation' one uses an integral transverse scale

$$L_{TT} = \int_0^\infty R_{TT}(y)\mathrm{d}y.$$

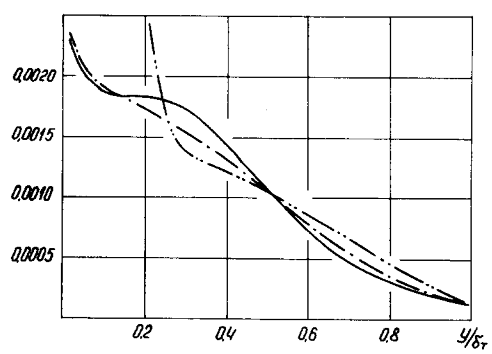

FIG. 4. Distribution of 'dissipation' of $\overline{v'\theta'}$.

———, experiment; —·—, $4\times10^{-3}\,\nu\frac{\overline{v'\theta'}}{\lambda_{TT}^2}$;

—··—, $0.12\,\nu\frac{\overline{v'\theta'}}{L_{TT}^2}$.

Then, as is seen from Fig. 4, an extremely high value of 'dissipation' is obtained in the wall region which cannot be attributed to the error due to the use of isotropic relations.

This shows the necessity of using, as is supposed in [11, 12], not only the integral, but also the dissipative scales to construct mathematical models for calculation of the turbulent boundary layer, which are based on the use of the turbulent heat flux transport equation.

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ETUDE EXPERIMENTALE DES COMPOSANTES DU FLUX THERMIQUE TURBULENT DANS LA SECTION DROITE D'UNE COUCHE LIMITE TURBULENTE RETARDEE

Résumé — On présente l'étude expérimentale de la distribution des termes de l'équation du transport turbulent de la chaleur dans la section transversale d'une couche limite turbulente dont l'équilibre est retardé. On fait la comparaison des termes de l'équation avec les approximations connues.

EINE EXPERIMENTELLE STUDIE ÜBER DIE KOMPONENTEN DES TURBULENTEN WÄRMESTROMGLEICHGEWICHTS IM QUERSCHNITT EINER VERZÖGERTEN TURBULENTEN GRENZSCHICHT

Zusammenfassung—Es werden Versuchsergebnisse über die Verteilung der Terme der turbulenten Transportgleichung des Wärmestroms im Querschnitt einer im Gleichgewicht befindlichen verzögerten turbulenten Grenzschicht angegeben. Es wird ein Vergleich der einzelnen Terme der Gleichung mit bekannten Näherungen angestellt.

ЭКСПЕРИМЕНТАЛЬНОЕ ИССЛЕДОВАНИЕ СОСТАВЛЯЮЩИХ БАЛАНСА ТУРБУЛЕНТНОГО ТЕПЛОВОГО ПОТОКА В СЕЧЕНИИ ЗАТОРМОЖЕННОГО ТУРБУЛЕНТНОГО ПОГРАНИЧНОГО СЛОЯ

Аннотация — В статье представлены экспериментальные данные по распределению членов транспортного уравнения турбулентного теплового потока в сечении равновесного заторможенного турбулентного пограничного слоя. Проведено сравнение отдельных членов уравнения с известными аппроксимациями.